Graph Theoretical Properties of Logic Based Argumentation Frameworks

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Extended Abstract

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ABSTRACT

Argumentation frameworks instantiated from logical language allow for argument generation over real knowledge. We present some graph theoretical properties of argumentation graphs obtained from an inconsistent knowledge base expressed using existential rules.

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1 RESEARCH CONTEXT

The contribution of this paper is to provide a complete characterisation of argumentation frameworks constructed from KBs solely composed of factual knowledge and negative constraints. This result can be used for designing argumentation solvers [3] that perform better in the presence of symmetries [6] or for generating realistic benchmarks [7]. In this short paper, for lack of space, we illustrate the contribution of the paper via a running example. We also focus on the simple case where the knowledge base does not contain any positive rules. Negative rules, expressing incompatibilities, are allowed. Please refer to [8] for the formalisation of the paper results.

Let us consider a knowledge base composed of four facts representing a possible instance of the famous Incompatible Food Triad where one is requested to find three foods (or drinks) for which any pair will taste good together, but all three together will not. The triad Beer, 7Up, and Whiskey was given as a solution with the statement that beer with 7Up makes *Shandy*, beer with whiskey makes a *boilermaker* (beer cocktail) and whiskey with 7Up is a *7 and 7*, but the three together would "make you sick".

Therefore, we cannot have at the same time a 7Up, a whiskey and a beer in a single cup (this is a negative constraint). Since the negative constraint is applicable to the facts, this knowledge base is inconsistent. The arguments generated over this knowledge base (there are 13 of them) are composed of a support and a conclusion, such that the conclusion is logically entailed by the support. The arguments are represented in Table 1.

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a_1	$(\{whiskey(ssc)\}, \{whiskey(ssc)\})$
a_2	$(\{soda(7Up)\}, \{soda(7Up)\})$
a_3	$(\{whiskey(ssc), soda(7Up)\}, \{whiskey(ssc), soda(7Up)\})$
a_4	$(\{beer(lb)\}, \{beer(lb)\})$
a_5	$(\{whiskey(ssc), beer(lb)\}, \{whiskey(ssc), beer(lb)\})$
a_6	$(\{soda(7Up), beer(lb)\}, \{soda(7Up), beer(lb)\})$
a_7	$(\{cup(c)\}, \{cup(c)\})$
a_8	$(\{whiskey(ssc), cup(c)\}, \{whiskey(ssc), cup(c)\})$
a_9	$(\{soda(7Up), cup(c)\}, \{soda(7Up), cup(c)\})$
a_{10}	$(\{whiskey(ssc), soda(7Up), cup(c)\}, \{whiskey(ssc), soda(7Up), cup(c)\})$
a_{11}	$(\{beer(lb), cup(c)\}, \{beer(lb), cup(c)\})$
a_{12}	$(\{whiskey(ssc), beer(lb), cup(c)\}, \{whiskey(ssc), beer(lb), cup(c)\})$
a_{13}	$(\{soda(7Up), beer(lb), cup(c)\}, \{soda(7Up), beer(lb), cup(c)\})$

Table 1: Arguments of the running example

The inconsistency is captured via the attacks that are defined as follows: an argument a_i attacks an argument a_j if and only if the conclusion of a_i with only one element of the support of a_j is sufficient to trigger a negative constraint. For example, a_3 attacks argument a_5 because the conclusion of a_3, $\{whiskey(ssc), soda(7Up)\}$, is inconsistent with $beer(lb)$ of the support of a_5.

The set of arguments and attacks can be visually represented using a directed graph where the nodes are labeled with the arguments and the attack relation is depicted via the edges. In Figure 1 below, the argumentation graph of this knowledge base is shown.

2 STRUCTURAL PROPERTIES OF ARGUMENTATION GRAPHS

The structural properties we propose in this paper enable us to have a general grasp of the layout of the generated graph. Indeed, we show the existence and describe three kinds of strongly connected components and some repetitive patterns in such graphs. Furthermore, we explicit the exponential number of arguments with respect to specific facts in the knowledge base. These structural properties are details as follows:

- **Number of isolated nodes** (nodes that are not attacked and that do not attack other arguments). This number is linked to the number of facts that are not in any minimal inconsistent set (free facts). A minimal inconsistent set is

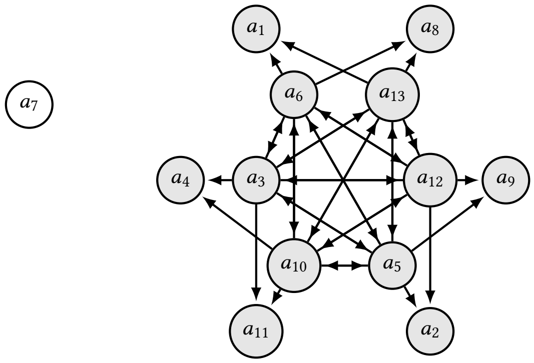

Figure 1: Argumentation framework corresponding to the running example

an inconsistent subset of facts of the knowledge base that becomes consistent as soon as at least one element is removed. We showed in [8] that the number of isolated nodes is exactly equal to $2^k - 1$ where k is the number of free facts. This property is responsible for the exponential explosion of the graph size as seen in [9]. For instance, in our running example, there is only one minimal inconsistent set $(\{whiskey(ssc), soda(7Up), beer(lb)\})$ and thus only one free fact $(cup(c))$. Therefore, we conclude that there is one isolated node (a_7) as represented in Figure 1. Please note that if we add two more free facts to the running example, we now have 55 arguments and 576 attacks.

- **Presence of repetitive patterns in subgraphs**. A k-copy graph of a graph G is a graph G' with k times more arguments. Each argument is copied k times and the attacks in G' satisfy (1) if a attacks b in G then a attacks all the k copies of b in G' and (2) if c is a copy of a in G' then c has the same attackers and attacks the same arguments as a. For instance, let G be the graph with six arguments $(b_1, \ldots, b_6)$ represented in Figure 2. The graph G' composed of the arguments $\{a_1, \ldots, a_6, a_8, \ldots, a_{13}\}$ (the grey nodes in Figure 1) and the corresponding attacks is a 2-copy graph of G.

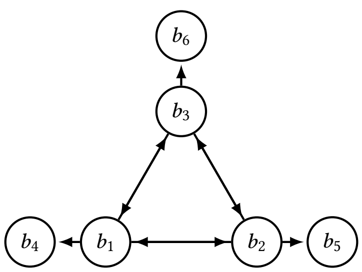

Figure 2: A graph G with six arguments and nine attacks

Given an argumentation graph with isolated nodes (thus coming from a knowledge base $\mathcal{K}$ with at least one free facts), the graph G obtained by removing the isolated nodes has the property that it is a 2^k-copy graph of the graph obtained from the previously mentioned knowledge base $\mathcal{K}$ without free facts where k is the number of free facts [8]. For example, if we remove the free fact $cup(c)$, we obtain the argumentation graph of Figure 2. Since we have one free fact, we know that this graph is a 2-copy graph of the subgraph in grey in Figure 1.

- **Strongly connected components (SCCs)**. In a directed graph a SCC is a subgraph such that for every two arguments a, b of this subgraph, there is a sequence of attacks (or path) from a to b. There are three kinds of SCCs in the argumentation graphs of [8]. The first kind is the isolated nodes. By definition, each isolated node is a SCC by itself since it does not attack any other argument. The second kind of SCCs is composed of arguments of the form (X, X) such that, for all minimal inconsistent sets of facts C, X does not contain a subset of C of size $|C - 1|$ (this represent the arguments that are attacked but do not attack other arguments). For example, the argument a_{11} in Figure 1 does not contain any subset of size 2 of the only minimal inconsistent set $\{whiskey(ssc), soda(7Up), beer(lb)\}$. Therefore, it is attacked by a_{10} and a_3 but it does not attack other arguments. It is a SCC by itself. Last but not least, all the other arguments that are not in the two aforementioned classes represent a SCC where each argument attacks all the other arguments. For example, the set of arguments $E = \{a_3, a_5, a_6, a_{10}, a_{12}, a_{13}\}$ is such that for every $a_i, a_j \in E, i \neq j$ we have $a_i \leftrightarrow a_j$.

Please note that the investigation of structural results of argumentation graphs computed over existential rules is of practical interest. Indeed, a similar instantiation [2] was used in the domain of food science in order to explain to non experts the outcome of inconsistent tolerant query answering results [1]. Based on the logical equivalence between repair semantics and argumentation semantics [4], the added value of argumentation over classical techniques was demonstrated in practice (see [5]).

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